Access to Science, Engineering and Agriculture: Mathematics 1 MATH00030 Chapter 2 Solutions

1. (a) In this case the slope of the line is $m = \dfrac{0-6}{0-(-3)} = \dfrac{-6}{3} = -2$. Thus the equation of the line is $y = -2x + c$, where we still have to find c. If we substitute $x = 0$ and $y = 0$ into $y = -2x + c$, we obtain $c = 0$. Hence the equation of the line is $y = -2x$.
   Note we could also say that $c = 0$ since the y-intercept is zero (since $(0, 0)$ is on the line).

   (b) In this case the slope of the line is $m = \dfrac{-1-5}{-1-3} = \dfrac{-6}{-4} = \dfrac{3}{2}$. Thus the equation of the line is $y = \dfrac{3}{2}x + c$, where we still have to find c. If we substitute $x = -1$ and $y = -1$ into $y = \dfrac{3}{2}x + c$, we obtain $c = -1 + \dfrac{3}{2} = \dfrac{1}{2}$. Hence the equation of the line is $y = \dfrac{3}{2}x + \dfrac{1}{2}$.

   (c) Here we note that the x-coordinates of the points are the same. Thus the line is parallel to the y-axis and its equation is $x = 5$.

   (d) Here we note that the y-coordinates of the points are the same. Thus the line is parallel to the x-axis and its equation is $y = 1$.

   (e) In this case the slope of the line is $m = \dfrac{1-(-2)}{-2-3} = \dfrac{3}{-5} = -\dfrac{3}{5}$. Thus the equation of the line is $y = -\dfrac{3}{5}x + c$, where we still have to find c. If we substitute $x = 3$ and $y = -2$ into $y = -\dfrac{3}{5}x + c$, we obtain $c = -2 + \dfrac{3}{5}(3) = \dfrac{-10+9}{5} = -\dfrac{1}{5}$. Hence the equation of the line is $y = -\dfrac{3}{5}x - \dfrac{1}{5}$.

2. (a) Here our line is parallel to a line that has slope -1, so our line also has slope $m = -1$. Hence the equation of the line is $y = -x + c$, where we still have to find c. On substituting $x = 6$ and $y = 3$ into $y = -x + c$, we obtain $c = 3 + 6 = 9$. Hence the equation of the line is $y = -x + 9$.

   (b) Here our line is parallel to a line that has slope 4, so our line also has slope $m = 4$. Hence the equation of the line is $y = 4x + c$, where we still have to find c. On substituting $x = -5$ and $y = 4$ into $y = 4x + c$, we obtain $c = 4 - 4(-5) = 24$. Hence the equation of the line is $y = 4x + 24$.
   Note the fact that the line $y = 4x + 123456789$ has y-intercept 123456789 has no bearing on this problem, the only relevant fact is its slope.

   (c) We know that the line $y = 5$ is parallel to the x-axis, so we are looking for the line through the point $(1, 2)$ parallel to the x-axis, which is $y = 2$.

3. (a) Since the line is parallel to the line through the points $(1, 3)$ and $(3, -5)$, it has the same slope. Now the slope of the line through the points $(1, 3)$ and

$(3,-5)$ is $\dfrac{-5-3}{3-1} = \dfrac{-8}{2} = -4$. Thus the equation of the line is $y = -4x + c$, where we still have to find c. Substituting $x = 3$ and $y = 1$ into $y = -4x + c$ we obtain $c = 1 + 4(3) = 13$. Hence the equation of the line is $y = -4x + 13$.

(b) Since the line is parallel to the line through the points $(-2,-3)$ and $(6,7)$, it has the same slope. Now the slope of the line through the points $(-2,-3)$ and $(6,7)$ is $\dfrac{7-(-3)}{6-(-2)} = \dfrac{10}{8} = \dfrac{5}{4}$. Thus the equation of the line is $y = \dfrac{5}{4}x + c$, where we still have to find c. Substituting $x = -1$ and $y = -3$ into $y = \dfrac{5}{4}x + c$ we obtain $c = -3 - \dfrac{5}{4}(-1) = -\dfrac{7}{4}$. Hence the equation of the line is $y = \dfrac{5}{4}x - \dfrac{7}{4}$.

(c) Here we note that the line through the points $(4,5)$ and $(4,7)$ is parallel to the y-axis (the two points have the same x-coordinate) so this is also true of the line we want. Since $(-1,-3)$ has x-coordinate -1, the equation we want is $x = -1$.

4. (a)

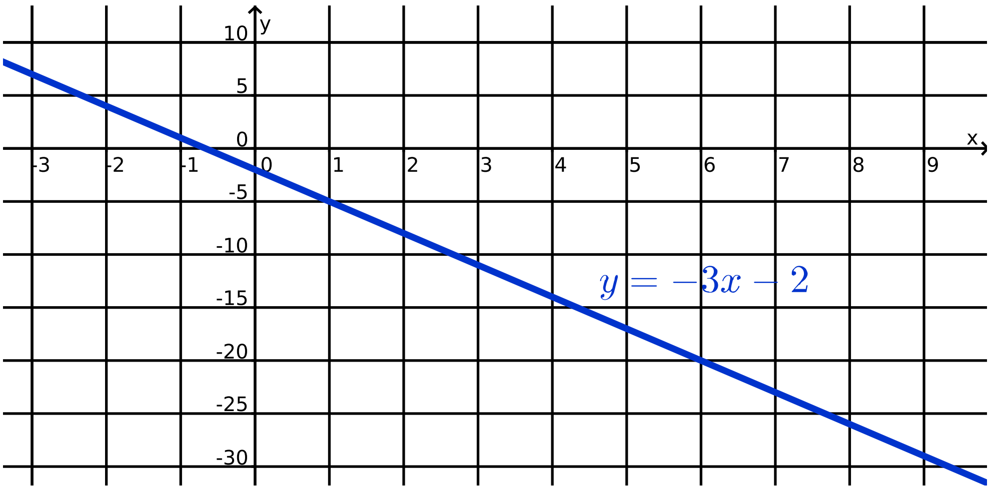

Figure 1: Graph of the line $y = -3x - 2$.

(b)

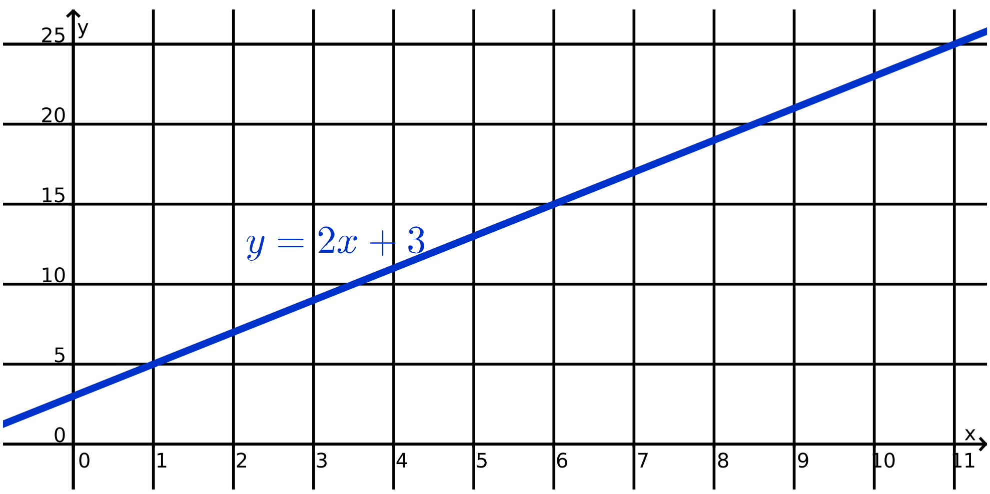

Figure 2: Graph of the line $y = 2x + 3$.

5. In the answers to these questions, I will alternate between the two methods described in the course notes.

   (a) From (1) we obtain $y = -3x + 4$ and on substituting this into (2) we obtain

   $$-2x + 3(-3x + 4) = 1 \Rightarrow -2x - 9x + 12 = 1 \Rightarrow -11x = -11 \Rightarrow x = 1.$$

   If we substitute this into $y = -3x + 4$, we obtain $y = -3(1) + 4 = 1$. Thus the solution is $x = 1$ and $y = 1$.

   (b) If we subtract two times (4) from (3) we obtain

   $$\begin{array}{rrcrcr} & -4x & + & 3y & = & 13 \\ - & -4x & + & -6y & = & -14 \\ \hline & & & 9y & = & 27 \end{array}$$

   Hence $y = 3$ and on substituting this into (3) we get $-4x = -3(3) + 13 = 4$, so that $x = -1$. Thus the solution is $x = -1$ and $y = 3$.

   (c) From (5) we obtain $2x = 5y + 18$, so that $x = \frac{5}{2}y + 9$. If we substitute this into (6) we obtain

   $$-3\left(\frac{5}{2}y + 9\right) - 4y = -4 \Rightarrow -\frac{15}{2}y - 27 - 4y = -4 \Rightarrow -\frac{23}{2}y = 23 \Rightarrow y = -2.$$

   Substituting $y = -2$ into $x = \frac{5}{2}y + 9$ we then get $x = \frac{5}{2}(-2) + 9 = 4$. Thus the solution is $x = 4$ and $y = -2$.

   (d) If we add seven times (8) to three times (7) we obtain

   $$\begin{array}{rrcrcr} & 21x & + & -6y & = & -57 \\ + & -21x & + & -35y & = & 98 \\ \hline & & & -41y & = & 41 \end{array}$$

   Hence $y = -1$. If we then substitute this into (7) we obtain $7x = 2(-1) - 19 = -21$, so that $x = -3$.
   Thus the solution is $x = -3$ and $y = -1$.

   (e) From (9) we obtain $2x = -3y + 7$, so that $x = -\frac{3}{2}y + \frac{7}{2}$. If we substitute this into (10) we get

   $$-6\left(-\frac{3}{2}y + \frac{7}{2}\right) - 9y = 8 \Rightarrow 9y - 21 - 9y = 8 \Rightarrow -21 = 8.$$

   Since $-21 \neq 8$ this means that there are no solutions.

   (f) If we add (12) to two times (11) we obtain

   $$\begin{array}{rrcrcr} & 4x & + & -2y & = & 8 \\ + & -4x & + & 2y & = & -8 \\ \hline & & & 0 & = & 0 \end{array}$$

   Here we have obtained $0 = 0$, so this means there are infinitely many solutions (note that (12) is a multiple of (11)). Since (11) yields $y = 2x - 4$, all the solutions are of the form $x = t$ and $y = 2t - 4$ where t is any real number.

6. (a) Using $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 2)$, the formula tells us that the length of the line segment is

$$\sqrt{(2-0)^2 + (2-0)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}.$$

(b) Using $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (-4, 2)$, the formula tells us that the length of the line segment is

$$\sqrt{(-4-(-2))^2 + (2-(-3))^2} = \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29}.$$

(c) Using $(x_1, y_1) = (2, -2)$ and $(x_2, y_2) = (-2, 2)$, the formula tells us that the length of the line segment is

$$\sqrt{(-2-2)^2 + (2-(-2))^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}.$$

(d) Using $(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (1, -2)$, the formula tells us that the length of the line segment is

$$\sqrt{(1-(-1))^2 + (-2-(-2))^2} = \sqrt{2^2 + 0^2} = \sqrt{2^2} = 2.$$

Note the y-coordinates of $(-1, -2)$ and $(1, -2)$ are the same, so the distance can also be calculated as $|1 - (-1)| = 2$.

7. (a) If we let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-3, 5)$, then using the formula, we have that the midpoint is

$$\left(\frac{0+(-3)}{2}, \frac{0+5}{2}\right) = \left(-\frac{3}{2}, \frac{5}{2}\right).$$

(b) If we let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (2, 3)$, then using the formula, we have that the midpoint is

$$\left(\frac{-1+2}{2}, \frac{2+3}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right).$$

(c) If we let $(x_1, y_1) = (2, -4)$ and $(x_2, y_2) = (2, 7)$, then using the formula, we have that the midpoint is

$$\left(\frac{2+2}{2}, \frac{-4+7}{2}\right) = \left(2, \frac{3}{2}\right).$$

Note the x-coordinates of $(2, -4)$ and $(2, 7)$ are the same, so the midpoint of $(2, -4)$ and $(2, 7)$ must also have this same x-coordinate.